

# $K^+$ 's Collective Flow in Heavy-ion Collisions predicted by Covariant Kaon Dynamics

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## Abstract

The directed and elliptic flows of positively charged Kaon produced in  $^{58}\text{Ni} + ^{58}\text{Ni}$  reaction at incident kinetic energy 1.91 AGeV, experimental data are released newly by V. Zinyuk, et.al. in Ref.[arXiv: 1403.1504v2 [nucl-ex] 8 Apr 2014], are reproduced by using the covariant kaon dynamics. Our numerical results indicate qualitatively the Lorentz force is necessary to explained reasonably the data as soon as the space-like part of kaon's vector potential is involved. The sensitivity of  $K^+$  directed as well as differential directed flow on the Lorentz force are also observed near target rapidity.

**PACS** number(s): 25.75.Ld, 24.10.Jv, 25.75.Dw

**Keyword:** Kaon meson; collective flow; heavy-ion collisions; Covariant dynamics.

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Kaon mesons production in heavy ion reactions at intermediate energies has been one of high interest topics in nuclear physics for several decades since it opens the possibility to attack several fundamental questions of nuclear and hadron physics which are not only interesting by themselves but have also astrophysical implications[1-6].  $K^+$  meson is produced in the early stage of heavy ion collisions at which the nuclear density in the reaction zone is much higher than the saturation density ( $\rho_0 = 0.16 fm^{-3}$ ). After its production  $K^+$  meson escapes near-freely from the reaction zone due to the relatively low  $K^+N$  scattering cross section ( $\sim 10$  milibarn ) and the absence of the absorption channel of a  $K^+$  meson on a nucleon in strong interaction. So, kaon production is proposed to be a sensitive probe to study the nuclear equation of state (EOS) in dense hadronic matter[7]. In 2002, P. Danielewicz et.al.[8] presented an EoS constrained by heavy ion flow data including Kaons has found wide-spread attention, meanwhile they also stated that it is impossible to find a unique formulation of the EoS that reproduces all the data. Other published experimental works on flow comparing data with various transport codes come to similar conclusions[9,10]: not only does one fail to reproduce all the data with a given code, there is unfortunately so far no really satisfactory agreement between the various theoretical approaches. In view of such situation, recently, W. Reisdorf and his Collaboration[11] started a large effort to complement earlier works by a systematic investigation encompassing a large range of system-energies using the large acceptance apparatus FOPI at the SIS accelerator. Most recently, they[12] reported on the simultaneous measurements of kaon and antikaon mesons in Ni + Ni collisions at an incident beam energy of 1.91AGeV that is close to the various strangeness production threshold energies. These newly-delivered experimental data provide us a chance to enrich our knowledge on the complicated dynamical processes and to improve the existed theoretical model. In practice, in Ref.[12] the authors have already compared the data to the predictions of some transport models for the azimuthal anisotropy of  $K^-$  mesons in a wide range of rapidity and the centrality dependence of  $p_T$  differential flow of  $K^+$  mesons.

As have already known[13,14,15], the combination of Quantum Molecular Dynamics(QMD) with the covariant kaon dynamics, in which a Lorentz-like force can be derived for the kaon mesons inside the nuclear medium, is one of successful theoretical transport model for simulating the  $K^+$  mesons production in heavy ion collisions at the SIS energy

region. With this model the collective flow of  $K^+$  meson and some associated produced particles have been reproduced reasonably[15,16,17]. Naturally, a question arises that is whether the model could describe the collective flow of  $K^+$ 's produced in this reaction. Alternatively, can the covariant kaon dynamics model reproduce the newly released experimental data? Obviously, to answer this question make sense since the validity of the model can be checked by virtue of the new data. Therefore, we present here the theoretical calculation within the covariant kaon dynamics and the comparison of them with the data. To this end we first review briefly the model and then give the results and discussions.

In the QMD model[13,18,19] each nucleon is represented by a coherent state of the form (we set  $\hbar, c=1$ )

$$\psi(\vec{r}, \vec{p}_0, t) = \frac{\exp[i\vec{p}_0 \cdot (\vec{r} - \vec{r}_0)]}{(2\pi L)^{3/4}} e^{-(\vec{r} - \vec{r}_0)^2/4L}. \quad (1)$$

where  $\vec{r}_0$  is the time dependent center of the Gaussian wave packet in coordinate space. The width  $L$  is kept constant, which means that one does not allow the spreading of the wave function. Otherwise, the whole nucleus would spread in coordinate space as a function of time.  $L$  is set to be  $L=1.08 \text{ fm}^2$  corresponding to a root mean square radius of the nucleons of 1.8 fm. To keep the formulation as close as possible to the classical transport theory, one uses Wigner density instead of working with wave function. The wigner representation of our Gaussian wave packets obeys the uncertainty relation  $\Delta r_x \Delta p_x = \hbar/2$ .

The time evolution of the N-body distribution is determined by the motion of the centroids of the Gaussians  $(r_{i0}, p_{i0})$ , which are propagated by the Poisson brackets

$$\dot{\vec{p}}_{i0} = \{\vec{r}_{i0}, H\} \quad \dot{\vec{r}}_{i0} = \{\vec{p}_{i0}, H\}. \quad (2)$$

Where  $H$  is the nuclear Hamiltonian

$$H = \sum_i \sqrt{\mathbf{p}_{i0}^2 + m_i^2} + \frac{1}{2} \sum_{i \neq j} (U_{ij}^{str} + U_{ij}^{cou}). \quad (3)$$

here  $U_{ij}^{str}$  is the nuclear mean field,  $U_{ij}^{cou}$  is the Coulomb interaction.

The natural framework to study the interaction between pseudoscalar mesons and baryons at low energies is the chiral perturbation theory (ChPT). From the chiral La-

grangian the field equations for the  $K^\pm$ -mesons are derived from the Euler-Lagrange equations [13,19]

$$\left[ \partial_\mu \partial^\mu \pm \frac{3i}{4f_\pi^{*2}} j_\mu \partial^\mu + \left( m_K^2 - \frac{\Sigma_{KN}}{f_\pi^{*2}} \rho_s \right) \right] \phi_{K^\pm}(x) = 0 \quad . \quad (4)$$

Here the mean field approximation has already been applied. In Eq. (4)  $\rho_s$  is the baryon scalar density,  $j_\mu$  is the baryon four-vector current,  $f_\pi^*$  is the in-medium pion decay constant. Introducing the kaonic vector potential

$$V_\mu = \frac{3}{8f_\pi^{*2}} j_\mu, \quad (5)$$

Eq. (4) can be rewritten in the form[21]

$$\left[ (\partial_\mu \pm iV_\mu)^2 + m_K^{*2} \right] \phi_{K^\pm}(x) = 0 \quad . \quad (6)$$

Thus, the vector field is introduced by minimal coupling into the Klein-Gordon equation. The effective mass  $m_K^*$  of the kaon is then given by[21,22]

$$m_K^* = \sqrt{m_K^2 - \frac{\Sigma_{KN}}{f_\pi^{*2}} \rho_s + V_\mu V^\mu} \quad . \quad (7)$$

where  $m_K = 0.496$  GeV is the bare kaon mass. Due to the bosonic character, the coupling of the scalar field to the mass term is no longer linear as for the baryons but quadratic and contains an additional contribution originating from the vector field. The effective quasi-particle mass defined by Eq. (7) is a Lorentz scalar and is equal for  $K^+$  and  $K^-$ .

The  $K^\pm$  single-particle energy is expressed as

$$\omega_{K^\pm}(\mathbf{k}, \rho) = \sqrt{\mathbf{k}^{*2} + m_K^{*2}} \pm V_0 \quad (8)$$

where  $k^* = k \mp V$  is the kaon effective momentum,  $k_\mu = (k_0, \mathbf{k})$ ,  $V_\mu = (V_0, \mathbf{V})$ . The kaon vector field is introduced by minimal coupling into the Klein-Gordon with opposite signs for  $K^+$  and  $K^-$ .  $m_K^*$  is the kaon effective (Dirac) mass. The kaon (antikaon) potential  $U_{K^\pm}(\mathbf{k}, \rho)$  is defined as

$$U_{K^\pm}(\mathbf{k}, \rho) = \omega_{K^\pm}(\mathbf{k}, \rho) - \omega_0(\mathbf{k}), \quad (9)$$

where

$$\omega_0(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m_K^2}. \quad (10)$$

In nuclear matter at rest the spatial components of the vector potential vanish, i.e.  $\mathbf{V} = 0$ , and Eqs. (6) reduce to the expression already given in Ref.[20]. The kaon (antikaon) potential  $U_{K^\pm}(\mathbf{k}, \rho)$  then reduce to the form

$$U_{K^\pm}(\mathbf{k}, \mathbf{V} = 0, \rho) = \sqrt{\mathbf{k}^2 + m_K^2 - \frac{\Sigma_{KN}}{f_\pi^{*2}} \rho_s + V_0^2} \pm V_0 - \sqrt{\mathbf{k}^2 + m_K^2}. \quad (11)$$

The covariant equations of motion are obtained in the classical (testparticle) limit from the relativistic transport equation for the kaons which can be derived from Eqs. (6). They are analogous to the corresponding relativistic equations for baryons and read[13,21]

$$\frac{dq^\mu}{d\tau} = \frac{k^{*\mu}}{m_K^*}, \quad \frac{dk^{*\mu}}{d\tau} = \frac{k_\nu^*}{m_K^*} F^{\mu\nu} + \partial^\mu m_K^* \quad . \quad (12)$$

Here  $q^\mu = (t, \mathbf{q})$  are the coordinates in Minkowski space and  $F^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$  is the field strength tensor for  $K^+$ . For  $K^-$  where the vector field changes sign. The equation of motion are identical, however,  $F^{\mu\nu}$  has to be replaced by  $-F^{\mu\nu}$ . The structure of Eqs. (12) may become more transparent considering only the spatial components

$$\frac{d\mathbf{k}^*}{dt} = -\frac{m_K^*}{E^*} \frac{\partial m_K^*}{\partial \mathbf{q}} \mp \frac{\partial V^0}{\partial \mathbf{q}} \pm \frac{\mathbf{k}^*}{E^*} \times \left( \frac{\partial}{\partial \mathbf{q}} \times \mathbf{V} \right) \quad (13)$$

where the upper (lower) signs refer to  $K^+$  ( $K^-$ ). The term proportional to the spatial component of the vector potential gives rise to a momentum dependence which can be attributed to a Lorentz force, i.e. the last term in Eq. (13). Such a velocity dependent ( $\mathbf{v} = \mathbf{k}^*/E^*$ ) Lorentz force is a genuine feature of relativistic dynamics as soon as a vector field is involved.

For the nuclear forces we use the standard momentum dependent Skyrme interactions corresponding to a soft EOS (the compression modulus  $K$  is 200 MeV). For the determination of the kaon mean field we adopt the corresponding covariant scalar–vector description of the non-linear  $\sigma\omega$  model. Here we use the parametrization of Ref.[23,24] which corresponds to identical soft nuclear EOSs. The shift of the production thresholds of the kaons by the in-medium potentials is taken into account as described in[22,25] . The hyperon fields are scaled according to SU(3) symmetry.

The flow of  $K^+$  mesons presented in the Ref.[12] has already been simulated with Isospin Dependent Quantum Molecular Dynamics model (IQMD) and Hadron String Dynamics(HSD) transport model and a good agreement of the theoretical calculations with

the data seem to be obtained. Here we examine the consistency of the prediction of the covariant kaon dynamics with the data. Before looking at our calculated results with the model, it is worth to recall that there are some distinctions between this model and IQMD though both of them are based on the chiral mean field approximation for the kaon in-medium interaction. Firstly, the way of kaons evolution in nuclear medium is not the same, namely, in covariant kaon dynamics the motion of the kaon meson is governed by covariant dynamical equation mentioned above but in the IQMD model the kaon evolves under the static potential given in Equ.(11) and thus the corresponding motion is not covariant. this is the most important differences of these models. Secondly, the parametrization of kaon's effective field is distinct, too. In IQMD the KN interaction coefficient  $\Sigma_{KN} = 350\text{MeV}$  and the pion decay constant  $f_\pi = 0.93\text{MeV}$ . This scenario of the kaon's effective field parametrization is called the Ko and Li scheme(KLP) [20] which corresponds to the kaon potential  $U_{K^+}(\rho_0) \approx 20\text{MeV}$  at saturation nuclear density. However, in the covariant kaon dynamics, following Ref. [14], the Brown & Rho parametrization (BRP):  $\Sigma_{KN} = 450\text{ MeV}$ ,  $f_\pi^{*2} = 0.6f_\pi^2$  for the vector field and  $f_\pi^{*2} = f_\pi^2$  for the scalar part given by  $-\Sigma_{KN}/f_\pi^{*2}\rho_s$  are used. This scenario accounts for the fact that the enhancement of the scalar part using  $f_\pi^{*2}$  is compensated by higher-order corrections in the chiral expansion[2,4]. Up to saturation density the Brown and Rho potential is  $U_{K^+}(\rho_0) \approx 30\text{MeV}$ . Additionally, the other pronounced difference between them is the treatment for proton and neutron and the nuclear field, that is, in the former the proton and neutron are taken as different kind of nucleons then the isospin degree of freedom and the relevant symmetry energy are embed in the mean field of nucleons while in the later both of them are considered as a same kind of particles and thereby the symmetry energy is not considered. Keep these distinctions in mind we proceed to inspect the calculation results here.

In Fig.1 we depict the variation of  $K^+$ 's directed flow,  $v_1$ , with respect to the rapidity of kaon produced in  $^{58}_{28}\text{Ni} + ^{58}_{28}\text{Ni}$  collisions at 1.91 AGeV. In the figure the line with diamonds indicate the experiments data[12] and the curve with solid-circles(hollow-circles) stand for the prediction of the covariant kaon dynamical model with BRP(KLP). And the curve with solid-triangles(hollow-triangle) represent the results of the QMD model with static potentials of kaons, i.e., without the Lorentz forces, which is incovariant for kaon

motion equation with BRP(KLP).

Fig.1

From this figure we can see that (1) the covariant kaon dynamics can reproduce the data in which the Lorentz force(LF) play a essential role in determining the  $K^+$ 's directed flow no matter if the KLP or BRP parameterization scheme being taken into account in the covariant kaon dynamics. This feature indicates, as has concluded in Ref.[14] by evaluating the transverse flow  $\langle p_x/m_k \rangle$ , that the effect of the LF contribution in the covariant kaon dynamics pulls the kaons back to the spectator matter; (2) the covariant kaon dynamics with BRP parameterization give a more reasonable pattern of the  $K^+$ 's directed flow since there are discrepancies in the cases of BRP and KLP when a same evolution equation being taken. Comparatively, the results with BRP is slightly higher than that with KLP. In view of the distinction induced by various parameterization scenarios is subtle we omit the results calculated with KLP in following figures and concentrate our attention on the influence of Lorentz-like force.

Fig.2 is plot for the different directed flow of  $K^+$ 's in the reaction systems at impact parameters  $b = 4.54$  fm(peripheral) and  $b = 2.11$  fm(central). The dotted-curves with hollow-circles corresponding to the variation of the directed flow with respect to the transverse momentum  $P_t$  of the kaons produced in the reaction without the kaon final interaction being taken into account and the other indicators of the curves are the same as those appearing in Fig.1. The curves in the figure corresponding respectively to the calculations via the covariant kaon dynamics(with LF) and the evaluation without the kaon final interaction(without  $U_K$ ) are close to each other. This tell us that the Lorentz force counterbalance the repulsive interaction of the  $K^+$  meson coming from nuclear environment and then resulting in the predictions comparable to the data. Therefore, the Lorentz force, a genuine feature of the relativistic kaon dynamics as soon as a vector field being involved, is also a crucial ingredients in determining the  $K^+$ 's different directed flow.

Fig.2

The rapidity dependence of  $V_2$  for  $K^+$ 's is plotted in the Fig.3. The indicators of the curves are the same as figures 2. As shown in the Fig.3 of Ref.[12] by comparing the

predictions of IQMD and HSD, the out-plane-flow  $V_2$  is not sensitive to the kaon final state interaction. This property is also demonstrated in the results of covariant kaon dynamics model.

Fig.3

In summary, in the present work the directed and elliptic flows of positively charged Kaon produced in  $^{58}_{28}\text{Ni} + ^{58}_{28}\text{Ni}$  reaction at incident kinetic energy 1.91 AGeV are evaluated within the covariant kaon dynamics combining by the quantum molecular dynamics model. Our results show that it is necessary to include the Lorentz force for reproducing the newly-delivered experiment data. The sensitivity of  $K^+$  the directed as well as differential directed flow near target rapidity are observed by comparing the results obtained via the covariant kaon dynamics model in the cases of with or without kaon's final state potential including Lorentz force. And such sensitivity can be attributed to the presence of Lorentz force as soon as the space-like part of vector potential of kaon being involved.

This work is supported in part by the National Natural Science Foundation of China (NSFC) under Grant No. 11265013.

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Figure 1: the variation of  $K^+$ 's directed flow, with respect to the rapidity of kaon produced in  $^{58}_{28}\text{Ni} + ^{58}_{28}\text{Ni}$  collisions at incident energy 1.91 AGeV. theoretical results is obtained at impact parameter  $b = 3.9\text{fm}$ .

Figure 2: The different direct flow of  $K^+$ 's in  $^{58}_{28}\text{Ni} + ^{58}_{28}\text{Ni}$  collisions at 1.91 AGeV with impact parameters  $b = 4.95\text{fm}$ (peripheral) and  $b = 2.11\text{fm}$ (central).

Figure 3: Rapidity dependence of elliptic flow of  $K^+$ 's in comparison to the predictions of covariant kaon dynamics in the cases of with and without the Lorentz force.